Analysis of Dispersion and Damping of Ion Waves in Plasma

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Experimental results are presented on the dispersion and damping of ion waves having a frequency range extending up to the ion plasma frequency. It was found that the Landau damping rate increases exponentially when the frequency of the ion wave approaches the ion plasma frequency; while its phase velocity decreases slightly. The experimental results agree reasonably with previous theoretical predictions. The study indicates significant changes in Landau damping even with small variations in the wave velocity.

Introduction

Numerous investigations on the propagation of low frequency ion waves have been performed both experimentally and theoretically [1–3]. Many of these investigations were aimed at studying the ion wave propagation at frequencies "ω", around the ion plasma frequency "ω_{pi}". Some controversial results have been reported with an increase in velocity and propagation beyond the ion plasma frequency [4] or a decrease in velocity and cutoff before ω_{pi} is reached [5]. It has been suggested that the increase of velocity in [4] might be due to the presence of free streaming particles [6, 7, 8]. At frequencies "ω" of ion waves well below the ion plasma frequency ω_{pi}, the phase velocity is found to be frequency independent and given by the Tonks-Langmuir speed. At these frequencies, the damping is proportional to the neutral gas pressure and is therefore primarily caused by ion-neutral collisions. At frequencies approaching ω_{pi}, the damping is higher than expected from collisions, and the excess is attributed to ion Landau damping [9].

The present work deals with an experimental study of dispersion and damping of ion waves in a frequency range extending up to the ion plasma frequency ω_{pi} and comparison of the obtained results with previous theoretical predictions based on the fluid model.

Theoretical Background

The dispersion relation and the Landau damping rate of ion waves in a plasma with $T_i \ll T_e$ can be written as [10]

$$V_p = \frac{\omega}{K_r} = \left(\frac{k\,T_e}{m_i} \right)^{1/2} \left(\frac{1}{1 + K_r^2 / K_d^2} + \frac{3\,T_i}{T_e} \right)^{1/2}, \quad (1)$$

$$K_i = \sqrt{\frac{\pi}{8}}\, \omega \sqrt{\frac{m_i}{k\,T_e}} \left(\frac{T_e}{T_i} \right)^{3/2} \quad (2)$$

$$\cdot \exp\left[-\frac{1}{2} \left(\frac{T_e}{T_i} \frac{1}{1 + K_r^2 / K_d^2} + 3 \right) \right]$$

where K_r and K_i are the real and imaginary parts of the wave number, T_e and T_i are the temperature of the electron and ion gas, and m_i is the ion mass, k Boltzmann's constant and K_d the Debye wave number $(4\,\pi\,n\,e^2/ k\,T_e)^{1/2}$ where n is the number density of the electrons. When the wave number K_r is much smaller than the Debye wave number K_d (or $\omega \ll \omega_{pi}$), both ω/K_r and K_i/K_r are independent of the frequency, that is, there is no dispersion either in the velocity or in damping. But as the frequency ω approaches the ion plasma frequency ω_{pi} (or the wave length λ approaches to the Debye shielding length) the phase velocity V_p of the ion wave slows down because the screening of the ion density perturbation by electrons has a little effect. The ions are then subject only to their own pressure gradient, and the phase velocity is finally reduced to the ion thermal velocity. This decrease in the phase velocity enhances the damping of the wave. The number of particles effective for Landau damping increases as the wave velocity approaches the ion thermal velocity. Therefore, dispersive effects appear in both the phase velocity and the wave damping. Since the factor $(1 + K_r^2/K_d^2)^{-1}$ enters into the exponent of (2), the shielding effect of electrons has a greater influence on the wave damping than on the phase velocity when $T_i \ll T_e$.
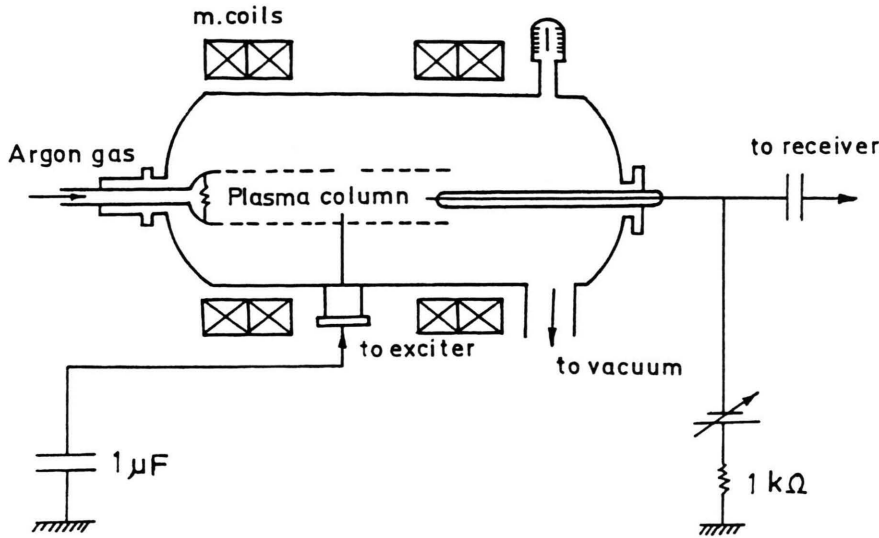
Fig. 1. Schematic diagram of the experimental apparatus.

If $T_i \ll T_e$ (cold plasma approximation), and if the effect of ion neutral collision is neglected, the waves are well described by the fluid theory which predicts a phase velocity given by [9]

$$V_p = \left[\frac{k\, T_e}{m_i} \left(1 - \frac{\omega^2}{\omega_{pi}^2} \right) \right]^{1/2}, \qquad (3)$$

which gives for $\omega \ll \omega_{pi}$ the Tonks-Langmuir velocity

$$V_s = \left(\frac{k\, T_e}{m_i} \right)^{1/2}.$$

The limiting value for the phase velocity V_p at high frequencies ($\omega > \omega_{pi}$) is given by the ion thermal velocity V_i as

$$V_i = \left(\frac{k\, T_i}{m_i} \right)^{1/2}. \qquad (4)$$

On the other hand, the collisional damping produces a second negative dispersion effect for $\omega \cong \nu_{in}$ where ν_{in} is the collision frequency for momentum transfer between ions and neutrals. At sufficiently high values of the ratio ν_{in}/ω_{pi}, this effect overlaps with the Landau damping and the Tonks-Langmuir velocity is not attained at any frequency for V_p. The lowering of the maximum in the combined dispersive curve is however only a few percent, as long as $(\nu_{in}/\omega_{pi}) < 1$.

It may be emphasized that the limiting value V_i and (3) are also found as simplified solutions of a kinetic treatment as given by Fried et al. [11] with the same assumptions $T_i \ll T_e$ and $\nu_{in} = 0$.

Experimental Arrangement

The experimental apparatus is rather simple, as shown schematically in Figure 1. The cylindrical vessel is 14 cm in diameter and 140 cm long. Argon gas is continuously bled through the system. The pressure (about 2×10^{-3} Torr) is controlled by a needle valve. The argon plasma is generated along the evacuated chamber parallel to a uniform magnetic field (1 kG). The created plasma forms a column, its diameter depending upon the precise value of the magnetic field, and is typically about 5 cm in diameter and 50 cm long. Langmuir probes are used to measure the electron temperature and plasma density. They are respectively about 2–3 eV and 10^8–10^9 cm^{-3}.

The experiment is carried out by applying an a · c voltage signal of 3 V$_{p-p}$ to a 0.25 mm diameter and 5 cm long negatively biased tungsten probe. The ion waves are detected by a negatively biased receiving probe for rejecting the electrons. The measurements have been done using the interferometer method [12], and indicated that $T_i \ll T_e$.
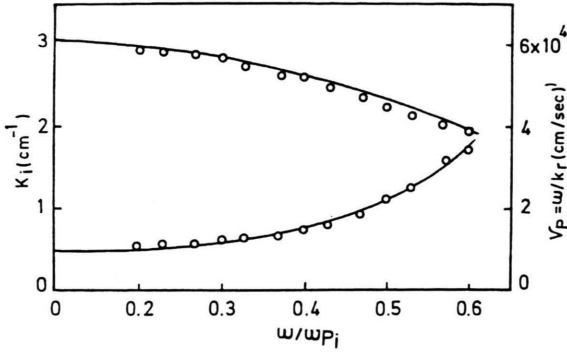
Fig. 2. Phase velocity "V_p" and damping rate K_i of ion wave against normalized frequency "ω/ω_{pi}". Damping includes both collisional and Landau damping.
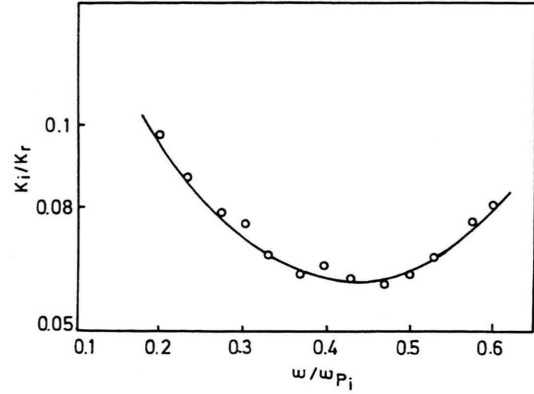

Fig. 4. Ratio of damping rate K_i to the wave number K_r i.e. K_i/K_r against ω/ω_{pi}.
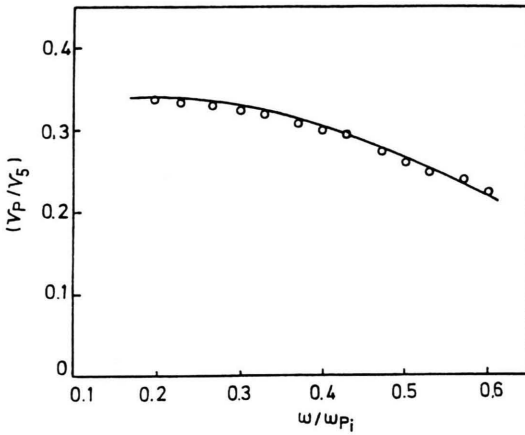

Fig. 3. Normalized velocity V_p/V_s against ω/ω_{pi}; 00: Experimental points. Solid line: Previous Theoretical Predictions based on the fluid model where $T_e = 10\ T_i$.
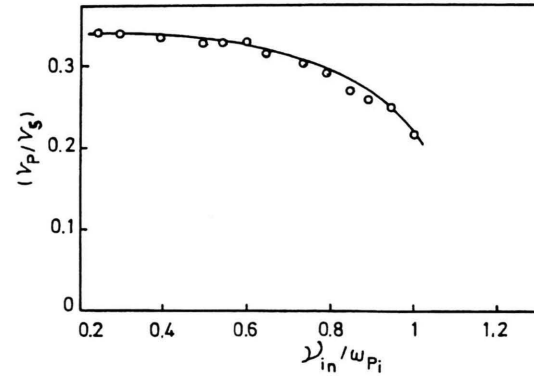

Fig. 5. Effect of the ratio (v_{in}/ω_{pi}) on the normalized velocity (V_p/V_s).

Results and Discussion

Figure 2 shows a typical example of the phase velocity V_p and damping rate K_i (for the propagation of ion waves) against the normalized frequency ω/ω_{pi}. It shows that velocity V_p decreases slowly as the frequency ω approaches the ion plasma frequency ω_{pi}. On the other hand, K_i increases exponentially as the frequency approaches ω_{pi}. Since the plasma is partially ionized, the collisional spatial damping occurs. The collisional damping rate is estimated by extrapolation of the damping curve to zero frequency. The corresponding collision frequency v_{in} can be obtained provided that the collisional damping rate K_i is expressed as $\dfrac{v_{in}}{2}\dfrac{K_r}{\omega}$ at $\omega \ll \omega_{pi}$ [9]. The solid lines in Fig. 2 are the theoretical

curves obtained from (1) and (2) with $T_e/T_i = 10$, where the estimated collisional damping rate has been added to the right hand side of (2), which agrees with the present results. At frequencies "ω" approaching "ω_{pi}", the negative dispersion effects due to Landau damping are presented in Figure 3. At lower frequencies the normal Tonks-Langmuir velocity V_s is obtained provided that v_{in} is sufficiently small, as mentioned above. The ratio of Landau damping rate K_i to the real wave number K_r is plotted against the normalized frequency ω/ω_{pi} in Figure 4. This is a normalized representation independent of the plasma density. The solid line in Fig. 4 is the previous theoretical prediction of the Landau damping calculated from (2) with $T_e = 10\ T_i$ which agrees with the present results. The normalized velocity (V_p/V_s) against the ratio v_{in}/ω_{pi} is given in Figure 5. It indicates that the

collisional damping produces a second negative dispersion effect for $\omega \cong v_{\text{in}}$. At sufficiently high values of the ratio $v_{\text{in}}/\omega_{\text{pi}}$, this effect overlaps with the Landau damping, and the Tonks-Langmuir value for the phase velocity is not attained at any frequency. The lowering of the maximum in the combined dispersive curve is, however, only a few percent as long as $v_{\text{in}}/\omega_{\text{pi}} < 1$ (Figure 5).

Conclusion

The dispersion and damping of ion waves with frequencies extending up to the ion plasma frequency has been studied experimentally. The results agree satisfactorily with previous theoretical predictions. At frequencies well below the ion plasma frequency ω_{pi}, the phase velocity V_{p} is found to be frequency independent and to be given by the Tonks-Langmuir speed V_{s}. At these frequencies the damping of the waves is caused by the ion-neutral collisions.

At frequencies approaching ω_{pi}, the damping is higher than expected from collisions, and the excess is attributed to Landau damping. This study of ion wave damping verified the dispersion of the Landau damping which becomes important even when there is a small decrease of the wave velocity.

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